

Exercise 55

For the following exercises, evaluate the expressions, writing the result as a simplified complex number.

$$\frac{(3+i)^2}{(1+2i)^2}$$

Solution

Simplify the given expression.

$$\begin{aligned}\frac{(3+i)^2}{(1+2i)^2} &= \left(\frac{3+i}{1+2i}\right)^2 \\ &= \left(\frac{3+i}{1+2i} \times \frac{1-2i}{1-2i}\right)^2 \\ &= \left[\frac{(3+i)(1-2i)}{(1+2i)(1-2i)}\right]^2 \\ &= \left(\frac{3-6i+i-2i^2}{1-2i+2i-4i^2}\right)^2 \\ &= \left(\frac{3-5i+2}{1+4}\right)^2 \\ &= \left(\frac{5-5i}{5}\right)^2 \\ &= (1-i)^2 \\ &= 1-2i+i^2 \\ &= 1-2i-1 \\ &= -2i\end{aligned}$$